APPROXIMATE CALCULATION OF PRESSURE DROP IN LAMINAR FLOW OF GENERALIZED NEWTONIAN FLUID THROUGH CHANNELS WITH INSERT

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An equation of Rabinowitsch–Mooney type has been suggested for approximate calculation of pressure drop in flow of generalized Newtonian fluid through channels with insert both in the region of creeping flow and at higher values of the Reynolds number, and this calculation method has been verified for four types of insert using own numerical solution and experimental results as well as literature data.

In practice, the calculation of flow of fluids through closed channels often encounters the problem of calculation of pressure drop in flow of generalized Newtonian fluid (GNF) through direct channels with insert. Channels with insert serve for heat transfer between two fluids (e.g., the heat exchanger of the tube-in-tube type), for intensification of heat transfer through the channel wall, or for mixing of fluids (static mixers).

Practically advantageous for calculation of pressure drop of channel are the Rabinowitsch–Mooney type equations which, using the most easily accessible solution for a Newtonian fluid (NF), enable also solution of the problem of calculation of pressure drop of an analogous flow of GNF. Their application is conditional on a satisfactory fulfilment of presumption of approximate agreement between the stress distribution in the flows of NF and GNF at the same value of pressure gradient, this agreement being exact for the channels of the simplest geometry (tube and planar slot).

The aim of the present work is to verify the applicability of the Rabinowitsch–Mooney type equation suggested in ref.[1] for the purposes of approximate calculation of pressure drop in creeping flow of GNF through channels of noncircular cross section without insert as well as for calculations of pressure drop in channels with insert even for higher values of the Reynolds number when the effect of inertial forces becomes significant during the flow.

THEORETICAL

The Rabinowitsch–Mooney type equation suggested in ref.[1] for the purposes of approximate calculation of pressure drop during flow through direct channels of noncircular cross section without insert reads as follows:

$$\dot{D}_w \equiv (3-\Omega)u_{ch}/l_{ch} = [(3+\Omega)/\tau_w^{(2+\Omega)}]\int_0^{\tau_w} \tau^{(1+\Omega)}\,\dot{D}(\tau)\,d\tau \qquad (1)$$

$$\tau_w = \Delta p l_{ch}/L \qquad (2)$$

$$u_{ch} = \dot{V}/S\ , \qquad (3)$$

where $\dot{D}_w$ means the kinematic consistency variable, τ_w means the dynamic consistency variable (Eq. (*2*)), u_{ch} is the characteristic (mean) velocity (Eq. (*3*)), l_{ch} is the characteristic linear dimension of system which is presumed to be independent of the flow properties of GNF, and $\dot{D}(\tau)$ is the dependence of deformation rate $\dot{D}$ on shear stress τ given by the general flow curve of GNF or by the respective flow model.

The dimensionless characteristic Ω of channel was expressed by relation[1]

$$\Omega = (r_h/r_p)^2\ , \qquad (4)$$

where $r_h = S/O$ is hydraulic radius, S is cross section of channel, O is its wetted perimeter, and $r_p = (S/4\pi)^{0.5}$ is the cross-sectional radius which is equal to the hydraulic radius of a tube of the same cross section S as has the channel considered.

Equation (*1*) is exactly valid for a tube of circular cross section ($l_{ch} = r_h \equiv D/4$; $r_h = r_p$; $\Omega = 1$) and for a planar slot ($l_{ch} = r_h \equiv h/2$; $r_p \to \infty$; $\Omega = 0$).

For a Newtonian fluid ($\dot{D}(\tau) = \tau/\mu$, where μ is the dynamic viscosity), integration of Eq. (*1*) results in relation (*5*), whereas for a power-law fluid ($\dot{D}(\tau) = (\tau/K)^{1/n}$, where K and n are parameters of the model) it results in relation (*6*).

$$\tau_w \equiv \Delta p l_{ch}/L = (3-\Omega)\mu u_{ch}/l_{ch} \qquad (5)$$

$$\tau_w \equiv \Delta p l_{ch}/L = K[(3-\Omega)(2+\Omega+1/n)/(3+\Omega)]^n (u_{ch}/l_{ch})^n \qquad (6)$$

Furthermore, in ref.[1] it was proved that application of more complex flow models to the case analyzed is unnecessary provided the parameters of power law flow model have been determined in the interval of shear stress τ or deformation rate $\dot{D}$ which correspond to the values of consistency variable τ_w or $\dot{D}_w$ attained during the flow through the channel given.

The pressure drop in the flow of GNF through a channel of noncircular cross section is determined from Eq. (*6*) using the characteristic linear dimension l_{ch} determined from the known solution for NF using Eq. (*5*) where the characteristic Ω is obtained from the channel geometry with the help of Eq. (*4*).

If Eqs (*1*) or (*6*) should be used for (even approximate) calculation of pressure drop in channels with insert, the quantities u_{ch}, l_{ch}, and Ω, necessary for the calculation, must be modified in the corresponding way. This is simple for such direct channels with inserts where neither the flowed-through cross section S_c nor the wetted perimeter O are significantly changed along the channel length.

For the channels with inserts of the above-mentioned type, which are considered in the present paper, it is

$$u_{ch} \equiv \dot{V}/S_c = \dot{V}/(S\varepsilon_s) \quad , \tag{7}$$

where S is the channel cross section with incorporated insert and

$$\varepsilon_s = S_c/S \tag{8}$$

is "surface" porosity of the channel-insert system. The value of "surface" porosity of the channel-insert system considered, which is denoted as ε below, is identical with the value of volume porosity,

$$\varepsilon_v = V_c/V = S_c L/(S\,L) \tag{9}$$

which can be made use of advantageously in its experimental determination.

From the geometry of flowed-through cross section it is also possible to obtain the values of hydraulic radius ($r_h = S\varepsilon/O$) and cross-sectional radius ($r_p = (S\varepsilon/4\pi)^{0.5}$) which are needed in calculation of Ω characteristic of channel with insert:

$$\Omega = 4\pi S\varepsilon/O^2 \quad . \tag{4a}$$

The characteristic linear dimension l_{ch} of channel-insert system is determined on the basis of known values of pressure drop Δp from Eq. (*5*), the pressure drop Δp, moreover, being involved in the momentum balance of fluid flowing through channel with insert in which the force effect of solid surface on fluid is expressed (like in immobile layer of particles[2]) as a sum of frictional and shape resistances.

On the basis of comparison of Eq. (*5*) with the momentum balance mentioned it can be expected that the ratio between the characteristic linear dimension l_{ch} and the usually adopted hydraulic diameter $D_h = 4r_h$ of channel without insert ($\varepsilon = 1$) can be expressed as

$$l_{ch}/D_h = f(\psi,\Pi_1,\Pi_2,\ldots) \quad , \tag{10}$$

where ψ is the ratio of shape and frictional resistances of channel with insert and Π_1, Π_2,…are simplexes of geometrical similarity of channel-insert system. The quantity ψ, which is considered independent of flow properties of GNF, has a constant value in the creeping flow region, its value being a function of the Reynolds number in the region of manifestation of inertial forces.

For a channel with insert of a given geometry (Π_1, Π_2, … = const.), the criterion dependence (*10*) can be replaced by Eq. (*11*), where Re_M is the Reynolds number of Rabinowitsch–Mooney type introduced into the treatment of momentum[2] by Eq. (*12*) in which ρ is density of the fluid.

$$l_{ch}/D_h = f(Re_M) \tag{11}$$

$$Re_M = \rho u_{ch}^2/\tau_w \tag{12}$$

The advantage of the Reynolds number defined in this way lies in its generality. With regard to validity of Eq. (*1*) for the flow of GNF, this criterion will assume approximately the same values independent of the GNF flow model used.

For a power-law fluid, introduction of τ_w from (*6*) into (*12*) will give the Reynolds number in the form:

$$Re_M = [(3-\Omega)(2+\Omega+1/n)/(3+\Omega)]^{-n}\ \rho u_{ch}^{(2-n)}\ l_{ch}^n/K \tag{13}$$

which is also valid for a Newtonian fluid for $n = 1$ and $K = \mu$.

However, this expression has a drawback in that it involves the linear dimension l_{ch} looked for. Therefore, for the purposes of calculation of pressure drop of channels with insert, the Reynolds number will be introduced by the relation

$$Re = (D_h/l_{ch})Re_M \qquad (14)$$

which for a power-law fluid assumes the form

$$Re = [(3 - \Omega)(2 + \Omega + 1/n)/(3 + \Omega)]^{-n} \rho u_{ch}^{(2-n)} l_{ch}^{(n-1)} D_h/K \qquad (15)$$

not any more involving the l_{ch} variable in the case of NF ($n = 1$), which simplifies the determination of concrete form of dependence (*11*) with the use of Eq. (*5*).

When calculating the pressure drop of power-law fluid, however, it is necessary to determine the l_{ch} value with the help of Eqs (*11*) and (*14*) by the method of gradual approximations. For the first approximation, one may choose the *Re* value at the limit of creeping flow region (D_h/l_{ch} = const.).

RESULTS AND DISCUSSION

In order to verify the suitability of the suggested way of calculation of pressure drop, we used the results of numerical solution for a tube with screw-shaped insert[3], experimental results for a channel with the cross section of annulus with double screw-shaped insert[4], experimental results in the creeping flow region for a tube with screw-shaped inserts[5], and results of numerical treatment for a channel with an insert of tube-in-tube type with the cross section of annulus[6].

The agreement between the pressure drop values Δp_{calc} calculated by the procedure given in Theoretical and the Δp values determined by some of the above-given procedures was evaluated according to the magnitude of mean quadratic deviation

$$\delta = [1/N \sum_{i=1}^{N} \delta_i^2]^{1/2} \ , \qquad (16)$$

where the per cent relative deviation δ_i is given as

$$\delta_i = (\Delta p/\Delta p_{calc} - 1) \ . \ 100\% \ . \qquad (17)$$

For the channel with screw-shaped insert (Fig. 1) we found at first the value of channel characteristic Ω given by Eq. (*4a*). As the insert divides the channel into two half-round parts with the same pressure drops, it is sufficient to consider only the flow in one of the two symmetrical parts of the channel. For the geometry considered in the numerical treatment[3] $s = 0$, $\varepsilon = 1$, $S = \pi D^2/8$, and $O = \pi D/2 + D$ we obtain the value $\Omega = 0.747$.

From the results of numerical treatment[3] for the flow of NF through the channel with this insert for the values of simplex D/L_e = 0.212, 0.260, 0.339, and 0.438, where D is inner diameter of tube and L_e is the length corresponding to rotation of thread of the screw insert by 180°, inclusive of the results for the channel of half-round cross section ($D/L_e = 0$), we obtained the dependence in the following form

$$D_h/l_{ch} = 6.13 + 2.89\,(D/L_e)^{2.02} + 0.0298\,(D/L_e)^{1.27}\,Re\ . \qquad (18)$$

The procedure of determining the dependence (*18*) with the help of data of pressure drop $\Delta p/L$ and the Reynolds number $Re_N = Du_{ch}\rho/\mu$ tabulated in ref.[3] for the flow of Newtonian fluid through screw-shaped inserts characterized by the above-given values of simplex D/L_e was as follows: First, the tabulated values of Re_N number were recalculated with the use of the definition equation (*15*), where $n = 1$ and $K = \mu$, to the values of Re number. Then on the basis of Eq. (*5*), the values l_{ch} and thereafter the values D_h/l_{ch} were determined for the corresponding $\Delta p/L$ data. Thus, altogether 46 triads of D_h/l_{ch}, D/L_e, Re data were available which were used for gradual construction of the dependence (*18*) with the help of linear regression. The validity of Eq. (*18*) is limited by the conditions $Re \le Re_{max}$ and $D_h/l_{ch} \le (D_h/l_{ch})_{max}$ which approximately delimit the region of stability of the numerical method used[3]. The values Re_{max}, $(D_h/l_{ch})_{max}$, and the

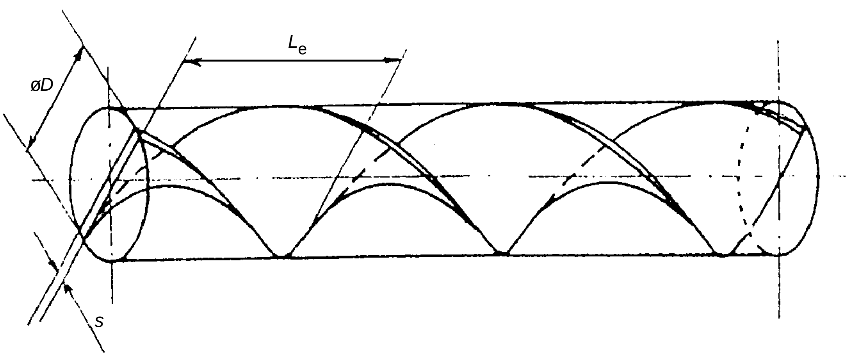

FIG. 1
Channel geometry of circular cross section with continuous insert

critical values Re_{cr} delimiting the creeping flow region (D_h/l_{ch} = const.) are given in Table I for the individual inserts.

The magnitude of δ_i deviations of the pressure drops Δp determined numerically from the Δp_{calc} values calculated from Eq. (*5*) with the use of Eq. (*18*) varied from –6.1% to 6.8%, the mean deviation δ given by Eq. (*16*) being 1.5%.

Next, for the values of flow index n = 0.9, 0.8, 0.7, 0.6, and 0.5, using Eq. (*18*) and the method of gradual approximation mentioned in Theoretical, we determined the values of pressure drop Δp_{calc} which were then compared with the corresponding values of numerical treatment[3]. Altogether 10 data were compared in the creeping flow region and 206 pressure drop data in the transition region.

A very good agreement was found with the compared values of pressure drop in the creeping flow region (D_h/l_{ch} = const., $Re \leq Re_{cr}$). The maximum value of relative deviation δ_i in this region, which corresponds to the flow index value n = 0.5 for a channel with the highest extent of vortex design of insert (D/L_e = 0.438) was only 1.5%.

With the channel of the same geometry with the ratio of D/L_e = 0.438 for the same flow index n = 0.5, we also found the absolutely highest value of deviation δ_i = –17.3%, namely for the maximum value of the Reynolds number Re_{max} = 532. In this case, of course, the value of pressure drop Δp determined at the limit of stability of numerical treatment can be loaded with a larger error of numerical method and, beside that, the calculated value of D_h/l_{ch} = 12.3 already exceeds the value 9.9 which delimits the validity of Eq. (*18*). For all the 206 data compared in the region of significant effect of Reynolds number, the mean deviation δ had the value of 3.9%. Hence, for the channel of the geometry considered, the calculation method of pressure drop suggested agrees well with the numerical treatment.

In order to verify the applicability of the suggested way of calculation of pressure drop we also used the experimental results for a channel of circular cross section with double screw-shaped insert[4] (Fig. 2). The insert is composed of two screw-arranged flat stripes with opposite direction of lead. The inner stripe of the height h_1 is firmly wound

TABLE I
Characteristics of channels with continuous screw-shaped insert (Fig. 1)

D/L_e	$(D_h/l_{ch})_{max}$	Re_{max}	Re_{cr}
0.212	11.0	1 097	0.60
0.260	8.9	488	0.62
0.339	9.9	502	0.63
0.438	9.9	323	0.64

on a tube or rod of diameter d coaxial with the channel. The outer stripe of the height h_2 is wound on the inner stripe and touches the inner wall of channel. The heights of stripes $h_1 = (d_1 - d)/2$ and $h_2 = (D - d_1)/2$ are chosen so as to obtain the same magnitudes of the areas of annuli (limited by the circles with diameters d, d_1 and D in Fig. 3) corresponding to the projections of screw-arranged stripes on the channel cross section. The contact points of the two stripes lie on two straight lines parallel with the channel axis.

Three types of insert were used whose geometrical characteristics (the ratio d/D of diameters of the outer and inner tubes, the ratio D/L_e of diameter of outer tube and distance of insert screw, and porosity of insert ε (determined by measuring the volumes V and V_c)[4]) are given in Table II.

Increasing complexity of the insert geometry increases also the difficulties connected with determination of value of the channel characteristic Ω. If the porosity of insert ε is determined experimentally, the magnitude S_c of the flowed-through cross section

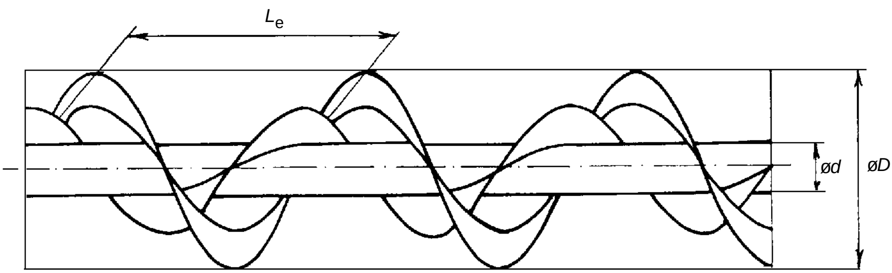

FIG. 2
Channel geometry of annular cross section with double screw-shaped insert

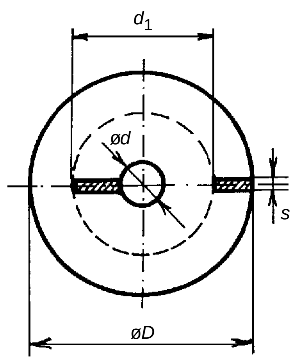

FIG. 3
Cross section of channel with double screw-shaped insert in Fig. 2

(which is needed for calculation of velocity u_{ch}) is simply obtained from Eq. (*8*) with the corresponding accuracy. On the other hand, the determination of wetted perimeter *O* of the channel is more difficult and may be accompanied by a larger error, which will then result in lowered accuracy of calculation of Ω characteristic according to Eq. (*4a*).

However, the accuracy of suggested way of calculation of pressure drop is not much affected by the error in determination of numerical value of Ω characteristic. For instance, in the discussed case of channel with screw-shaped insert (Fig. 1) with the characteristics $s = 0$ and $D/L_e = 0.438$, the deviation of pressure drop from the numerical treatment is 1.5% in the creeping flow region for the flow index value $n = 0.5$. If the correct value of $\Omega = 0.747$ in Eqs (*5*) and (*6*) is replaced by, e.g., $\Omega = 0.5$, the magnitude of this deviation will change from 1.5% to 2.1% only.

The geometry of cross section of channel with double screw-shaped insert with a half distance between two neighbouring contact points of the two stripes on the straight line is depicted in Fig. 3. For this cross section geometry, the magnitude of wetted perimeter can be determined from the relation:

$$O = \pi(D + d) + D - d \quad . \qquad (19)$$

The relation given can be used for any cross section except for those at the points of crossing of the stripes where the wetted perimeter is smaller. Obviously, the requirement of constant value of channel perimeter is fulfilled only partially with this insert.

TABLE II
Characteristics and experimental results for annulus with double screw-shaped insert (Fig. 2)

Insert	1	2	3
$D_h = D$, mm	40	40	40
d/D	0.130	0.156	0.204
D/L_e	2.78	1.68	1.35
ε	0.851	0.864	0.852
Ω	0.430	0.426	0.401
A	36.1	19.9	16.2
B	13.5	7.5	5.0
X	0.39	0.36	0.40
Y	6	6	6
δ_{NF}, % (N = 20)	2.0	2.2	2.3
δ_{GNF}, % (N = 25)	5.0	12.3	6.3

The values of Ω characteristic of the individual inserts calculated with application of the experimental porosity value and Eqs (*4a*) and (*19*) are given in Table II.

The further treatment of experimental data of pressure drop for NF in ref.[4] was similar to that given above with the only difference that, with regard to a larger number of dimensionless geometrical characteristics in the insert of this type, an individual relation of general form (*20*) was determined for each insert type,

$$D_h/l_{ch} = [A^Y + (B\,Re^X)^Y]^{1/Y} \;, \quad \text{for } Re < 70 \tag{20}$$

and the numerical values of coefficients *A, B, X, Y* obtained from experimental data by optimization are given in Table II for the individual types of inserts. For $N_{NF} = 20$ experiments with NF, with each individual insert, the mean deviation δ of experimental values of pressure drop from the values calculated with the use of Eqs (*20*) and (*5*) varied from 2.0% to 2.3% (see Table II).

The experimental results with NF and GNF ($N_{GNF} = 25$ for each individual insert) are given in Fig. 4 for the flow index range $0.47 < n < 0.59$ and for $Re > 0.8$. From Fig. 4 it can be seen that, in contrast to the numerical treatment of flow of GNF through the screw-shaped insert given in Fig. 1, the agreement of experimental results with GNF is better in the region of manifestation of the Reynolds number than in the creeping flow region, and the Re_{cr} values delimiting the creeping flow region can be estimated for the individual inserts on the basis of relations (*20*) from the condition $A >> B\,Re^X$.

The fact that positive deviations δ_i predominated for the inserts 1 and 3 in the creeping flow region whereas for insert 2 they were all negative has its reason obviously in imprecision of construction of the inserts affecting the system geometry, which is then also reflected in the resulting values of mean deviations δ (see Table II). The higher δ_i

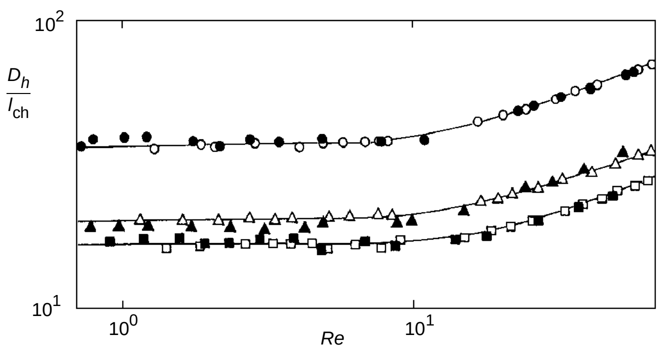

FIG. 4
Dependence of dimensionless ratio D_h/l_{ch} on the Reynolds number *Re* for channels as in Fig. 2: ❍ ● insert 1, Δ ▲ insert 2, ❐ ■ insert 3, empty points NF, full points GNF

values can partly be due to the fact that the flowed-through cross section with the double screw-shaped insert does not form a simply continuous area, which is contrary to the requirement for application of Eq. (*6*) to calculation of pressure drop[1] with regard to the presumption of agreement of distribution of stress during flow of NF and GNF. The maximum value of relative deviation δ_i was –14.9% here (insert No. 2, Re = 3.3).

The flow of GNF through channels depicted in Figs 1 and 5 was also studied in ref.[5]. The insert represented in Fig. 1 has a form of continuous screw surface, whereas the insert represented in Fig. 5 used for static mixers is composed of elements having shapes of screw surface (elements of Kenics type). The dextrorotatory and laevorotatory elements are installed alternately into the tube and the edges of neighbouring elements form an angle of 90°.

Both the inserts were characterized by the value of simplex of geometrical similarity s/D = 0.095, where s is thickness of the insert stripe. The experimental results with NF and GNF in the creeping flow region at the flow index values of $0.5 \le n \le 1$ led the authors[5] to derive the criterion dependences (*21*) and (*22*) for the inserts represented in Figs 1 and 5, respectively,

$$\Delta p D^{(n+1)}/(KLu^n) = [168.10 + 201.23(D/L_e)]n^{1.95} \tag{21}$$

$$\Delta p D^{(n+1)}/(KLu^n) = [187.60 + 196.84(D/L_e)]n^{1.80} \ , \tag{22}$$

where L_e is the length of insert element.

In accordance with the authors[5], the value of flowed-through cross section, whose geometry is represented in Figs 1 and 5, was calculated by relation (*23*), and the wetted perimeter of channel by relation (*24*).

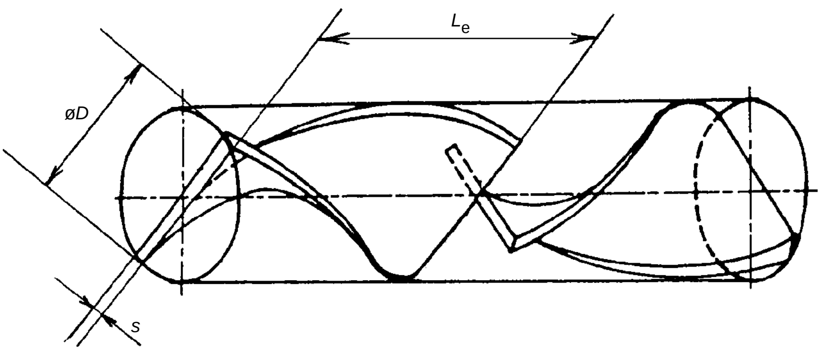

FIG. 5
Channel geometry of circular cross section with interrupted screw-shaped insert

$$S_c \equiv S\varepsilon = \pi D^2/4 - sD \tag{23}$$

$$O = \pi D + 2D - 2s \tag{24}$$

Hence, for the value of simplex of geometrical similarity $s/D = 0.095$ we get, using the relation (*4a*), the value $\Omega = 0.354$. In this context it must be noted that the relations (*23*) and (*24*) are not valid with insert represented in Fig. 5 for the cross sections at the contact points of individual elements: here the part of cross section assumed by the insert is almost double as compared with other cross sections, which is reflected not only in the local value of Ω characteristic but also in the value of mean velocity u_{ch}. Hence, the insert represented in Fig. 5 strictly fulfills neither the requirement of constant value of flowed-through cross section nor that of wetted perimeter of channel.

The values of D_h/l_{ch} quantity were calculated for the individual values of insert characteristics D/L_e given in Tables III and IV using the relation (*5*) and the relations (*21*) and (*22*) valid for NF ($n = 1$). The D_h/l_{ch} values obtained are presented in Tables III and IV, too. Furthermore, Eq. (*6*) was used to calculate the values of pressure drop Δp_{calc} for the minimum value of flow index $n = 0.5$. The Δp_{calc} values were then compared with the corresponding Δp values calculated from Eqs (*21*) and (*22*). The deviations found, δ and δ_i, are also given in Table III and IV.

For the insert represented in Fig. 1 in the creeping flow region (D_h/l_{ch} = const.), like for channels without insert[1], the absolute values of δ_i deviations from experimental values of pressure drop are higher than the deviation values from the pressure drops determined numerically; the maximum magnitude of value of relative deviation $\delta_i = 7.0\%$

TABLE III
Characteristics and comparison of own treatment with relation[5] for continuous screw-shaped insert (Fig. 1) and the flow index value $n = 0.5$

D/L_e	D_h/l_{ch}	δ_i, %
0.212	6.31	–7.0
0.260	6.45	–6.0
0.339	6.68	–4.3
0.439	6.96	–2.3
		$\delta = 5.2$

can, nevertheless, be considered satisfactory. As for the insert represented in Fig. 5, the periodical narrowing of cross section at the contact points of insert elements makes itself felt in an increase in pressure drop as compared with that of insert represented in Fig. 1, which is documented by results of Table IV. Here the deviations δ_i are positive but their magnitude is also acceptable.

A special group of channels with insert is represented by such channels in which the flowing around insert is only connected with the friction component of resistance. Their simplest representative is a channel of tube-in-tube type with annular cross section. The annulus geometry can be characterized by the ratio $k = d/D$, where d is the inner and D the outer diameters of the annulus.

However, even here it must be considered that the flowed-through cross section in the form of annulus does not form a simply continuous area according to the presumption. Hence it can be expected, like with the channel with double screw-shaped insert, that the Δp_{calc} values determined by the procedure suggested using Eq. (*6*) with the Ω value given by Eq. (*4a*) can be loaded with larger errors than those in the other cases.

Comparison of values of pressure drop Δp_{calc} calculated from Eq. (*6*) and the characteristic Ω given by Eq. (*4a*) with the Δp values[6] from numerical treatment for the flow of power-law fluid through a channel with annular cross section are given in Table V for selected values of flow index $0.1 \leq n \leq 1$. The D_h/l_{ch} values necessary for calculation Δp_{calc} were determined from the relation

$$D_h/l_{ch} = (32/\{(3 - \Omega)[1 + k^2 - (1 - k^2)/\ln k^{-1}]\})^{1/2} \qquad (25)$$

obtained by comparing Eq. (*5*) with analytical solution[7] of flow of NF through annulus.

A limit case of cross section of the annulus shape, which is considered a doubly continuous area, is infinite planar slot ($k \to 1$) with the value of characteristic $\Omega = 0$. Equations (*5*) and (*6*) are exactly valid for the planar slot.

TABLE IV
Characteristics and comparison of our treatment with relation[5] for interrupted screw-shaped insert (Fig. 5) and the flow index value $n = 0.5$

D/L_e	D_h/l_{ch}	δ_i, %
0.199	5.95	4.9
0.247	6.68	5.9
0.330	6.91	7.7
0.495	7.34	11.0
		$\delta = 7.7$

It is interesting to find that the prediction of pressure drop Δp_{calc} according to Eq. (*6*) with constant value of the characteristic $\Omega = 0$ and the thereto corresponding D_h/l_{ch} value (Eq. (*25*)) provides more precise results in a broad range of $0.01 \le k \le 1$, as compared with the previous procedure.

This is obvious from Table V summarizing the δ_i values for both calculation procedures considered. Even at a very low value of the ratio $k = 0.01$, the relative deviation does not exceed 4.7% for $\Omega = 0$, and at $k \ge 0.2$ the values of deviations δ_i are below 1%.

CONCLUSION

A procedure has been suggested for approximate calculation of pressure drop in laminar flow of GNF through direct channels with inserts both in the creeping flow region and in the region of significant inertial forces.

For this purpose, presuming an agreement between distribution of stress during flow of NF and GNF characterized by the same value of $\Delta p/L$ ratio, the validity range has

TABLE V
Relative deviations δ_i of numerical calculation of pressure drop of channel of annular cross section[6] from the values calculated from Eq. (*6*)

k	n	Ω	δ_i, %	Ω	δ_i, %
0.01	1	0.980	0.0	0	0.0
	0.5		–8.5		–2.2
	0.25		–10.6		0.4
	0.1		–10.6		4.7
0.1	1	8.818	0.0	0	0.0
	0.5		–6.2		–1.2
	0.25		–9.3		–0.7
	0.1		–10.9		1.0
0.2	1	0.667	0.0	0	0.0
	0.5		–4.6		–0.7
	0.25		–7.3		–0.5
	0.1		–9.0		0.4
0.3	1	0.538	0.0	0	0.0
	0.5		–3.3		–0.4
	0.25		–5.6		–0.3
	0.1		–7.2		0.2

been extended of the Rabinowitsch–Mooney type equation which was suggested earlier for the purpose of calculation of pressure drop in channels without insert.

The applicability of the procedure suggested has been verified for a continuous screw-shaped insert, the insert composed of screw-shaped elements (Kenics type), double screw-shaped insert, and for the tube-in-tube geometry.

It has been documented that the given method of calculation of pressure drop agrees well with results of both numerical treatment and experiment for all four types of inserts studied.

SYMBOLS

A, B	coefficients in Eq. (*20*)
D	tube diameter, m
$\dot{D}$	deformation rate, s^{-1}
D_h	hydraulic diameter of channel, m
$\dot{D}_w$	consistency variable, Eq. (*1*), s^{-1}
d	diameter of inner tube
d_1	outer diameter of inner screw stripe and inner diameter of outer screw stripe in channel with double screw-shaped insert, m
h	width of slot, m
h_1	height of inner screw stripe in channel with double screw-shaped insert, m
h_2	height of outer screw stripe in channel with double screw-shaped insert, m
K	parameter of power-law model, Pa s^n
k	ratio of diameters of inner and outer tubes
L	channel length, m
L_e	length of insert element, m
l_{ch}	characteristic linear dimension of system, m
N	number of values compared
n	parameter of power-law model
O	channel perimeter, m
Δp	pressure drop, Pa
$r_h = D/4$	hydraulic radius of tube, m
$r_h = h/2$	hydraulic radius of slot, m
r_p	cross-sectional radius, m
S	cross section of channel without insert, m^2
S_c	free cross section of channel with insert, m^2
s	thickness of insert, m
u_{ch}	characteristic rate of system, Eq. (*7*), m s^{-1}
V	volume of channel without insert of finite length, m^3
$\dot{V}$	volume flow rate, m^3 s^{-1}
V_c	free volume of channel with insert of finite length, m^3
X, Y	coefficients in Eq. (*20*)
Π	simplex of geometrical similarity
Ω	shape characteristic of channel
δ	mean deviation
δ_i	relative deviation
ε	insert porosity

μ	dynamic viscosity, Pa s
ρ	density of fluid, kg m^{-3}
τ	shear stress, Pa
τ_w	consistency variable, Eq. (*2*), Pa
ψ	ratio of shape and frictional resistances of insert
Re_M	Reynolds number, Eqs (*12*) and (*13*)
Re	Reynolds number, Eqs (*14*) and (*15*)

Indexes

calc	calculated by the method suggested in this paper
cr	critical
max	maximum

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